A Mathematical Model to Represent Weightless Man

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AS SPACE operations progress, man will be required to perform many physical functions while weightless. These functions will include extra-vehicular activities such as spacecraft assembly, maintenance, supply, and personnel rescue. The astronaut, floating free from space vehicle, will experience degrees of freedom never encountered on earth or in a space capsule. While "situated in a state of imponderability,"[6] any force or torque applied by or to man will result in translational and/or angular accelerations or decelerations. Internal forces and torques will be generated and reacted throughout his body as he moves, and may produce angular motions.[5] To maneuver and work in space, the astronaut of the future must be equipped with a system to provide propulsion, stabilization, and life support.

The most desirable characteristics of this self-maneuvering and life support system are maximum freedom of motion for the man, and minimum mass for the system. These criteria make the system dynamics heavily dependent upon the dynamic response characteristics of the human body. A gap exists, however, between anthropometric data and dynamic parameters needed to design a self-maneuvering unit for the astronaut. A mathematical model which will represent the biomechanical properties of the human body is needed to bridge this gap. The purpose of this paper is to describe such a model. It is concerned with only those major dynamic effects which result when the human body is subjected to unbalanced forces, and not the physiological and psychological problems of manned space flight. The development and analysis of the model is followed by the description of an effort to validate the model experimentally.

THE MATHEMATICAL MODEL

The most important parameters (or biomechanical properties) which determine the body's dynamic response characteristics are total mass, location of the center of mass, and moments of inertia. When these properties are incorporated into the math model, the model can then be used to predict analytically how the body will respond.

Factors such as elasticity and damping of the body structure do not appear to have any significant effect on the design of the self-maneuvering unit and are not considered here.

To develop the math model, the human body is idealized in the following manner:

1. The human body is divided into a finite number of masses (or segments) which are considered to be

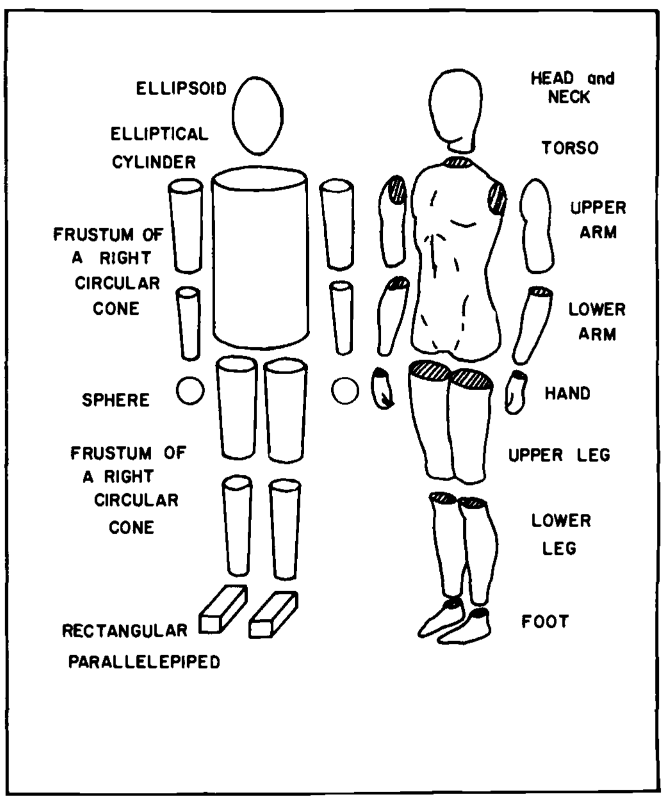

Fig. 1. Segmented man and model.

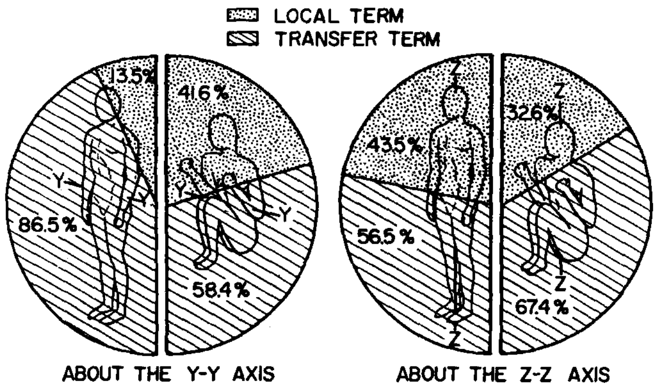

Fig. 2. Comparison of local to transfer moment of inertia terms (expressed as a per cent of the total moment of inertia).

rigid and homogeneous, and a finite number of degrees of freedom (or segment hinge points).

2. Each segment is represented by a geometric body which closely approximates the segment's shape, mass, mass center, length, and average density.

Thus, the model may be thought of as a system of rigid, homogeneous bodies of relatively simple geometric shape, hinged together in such a manner as to resemble the human body.

The segmentation of the body and the representative geometric bodies are shown in Figure 1 for a 14-segment model. The dimensions of the segments are based either on actual body measurements of a particular individual or on statistical anthropometric data. Because mass, center of mass, and density of the segments can not yet be determined accurately, this information is also based on statistical data.[1]

The variation of the center of mass of the human body has been studied extensively[3] and can be accurately predicted for a given body position without too much difficulty. The height of the model's center of mass is found to be 56.6 per cent of the stature. This falls within the 55 to 57.4 per cent range determined experimentally by Dempster[1] and agrees closely with an average of 55.6 per cent measured by Swearingen[7] on five living subjects.

Predicting the moments of inertia is somewhat more involved and likely to be less accurate. Therefore, the 14-segment model of Figure 1 was analyzed to determine:

1. which segments have the greatest effect on the total moment of inertia,
2. the effect of approximation errors due to representing the segments by geometric bodies,
3. and which segments can be further simplified without a significant loss in accuracy.

The moment of inertia of the whole body about a given axis is equal to the sum of the moments of inertia of all the segments about that axis. The moment of inertia of each segment is given by the parallel axes transfer equation

$$I = I_{c.g.} + md^2 \qquad (1)$$

where $I_{c.g.}$ ≡ Local Term = Moment of inertia of the

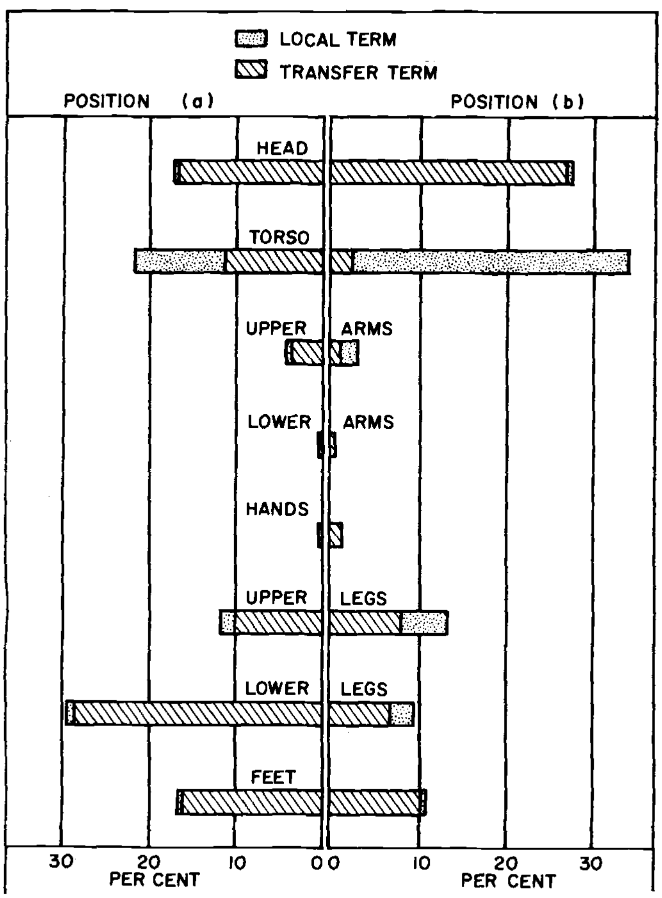

Fig. 3. Moment of inertia of each segment (expressed as a per cent of the body moment of inertia for the Y-Y axis).

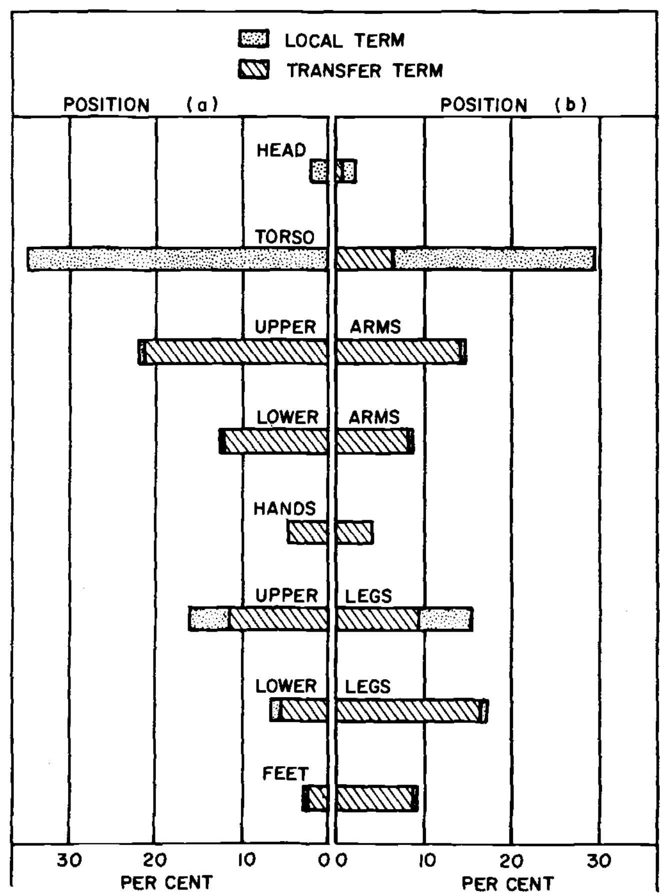

Fig. 4. Moment of inertia of each segment (expressed as a per cent of the body moment of inertia for the Z-Z axis).

segment about an axis through its mass center, $md^2 \equiv$ Transfer Term = Product of the segment mass (m) and the square of the prependicular distance (d) between parallel axes through the segment's mass center (same axis as for local term) and the whole body center of mass

First, it is of interest to compare the local and transfer terms for the whole body. In Figure 2, the local and transfer terms are expressed as percentages of the total body moment of inertia for the two positions and axes shown. It is readily apparent that the local terms play a significant part for many body positions. The methods of calculating local moments of inertia for the segments and the values used here are described.[8]

When the local terms are examined individually, however, some terms are found to be negligible. This can be seen from Figures 3 and 4. The positions "a" and "b" are shown schematically in Figure 5. Consequently, it is unnecessary to compute the local moments of inertia for the hands, lower arms, and feet because their sum is less than the errors caused by simplifying the human body. Furthermore, the geometric representation for the upper arms, upper legs, lower legs, and head need not be too accurate. For instance, a 33 per cent variation in the local moment of inertia of the upper arm would change the total moment of inertia (for the standing position) about the Y-axis only ± 0.1 per cent. The local moments of inertia of the frustums of right circular cones can be approximated within ± 5 per cent with right circular cylinders. However, care must be exercised to use the correct mass center location. The local moment of inertia of the torso must be computed more accurately because it may contribute 10 per cent to 35 per cent of the total moment of inertia depending on the axis and position.

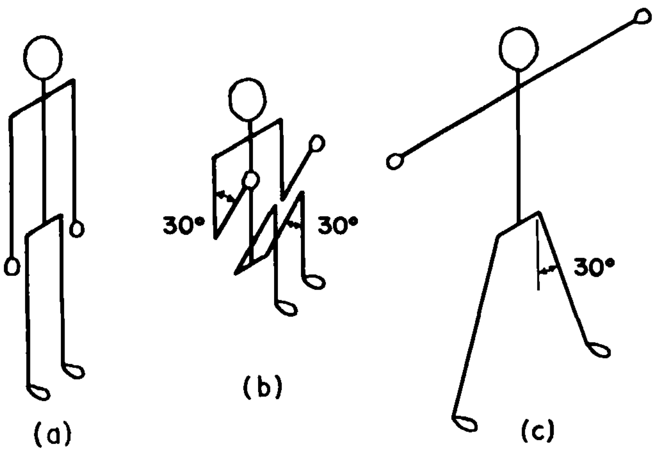

Fig. 5. Body positions.

Based on the above results, a simplified method is derived for computing the moments of inertia for various body positions. Starting with the moments of inertia for the standing position as initial conditions (Ix_0, Iy_0, Iz_0), this method yields the moments of inertia for any other position (Ix, Iy, Iz) by taking into account only the changes in the transfer terms and the relative position of the body axis system. This approach greatly simplifies the mathematics, and although it neglects the changes in the local terms, there is only a slight reduction in accuracy.

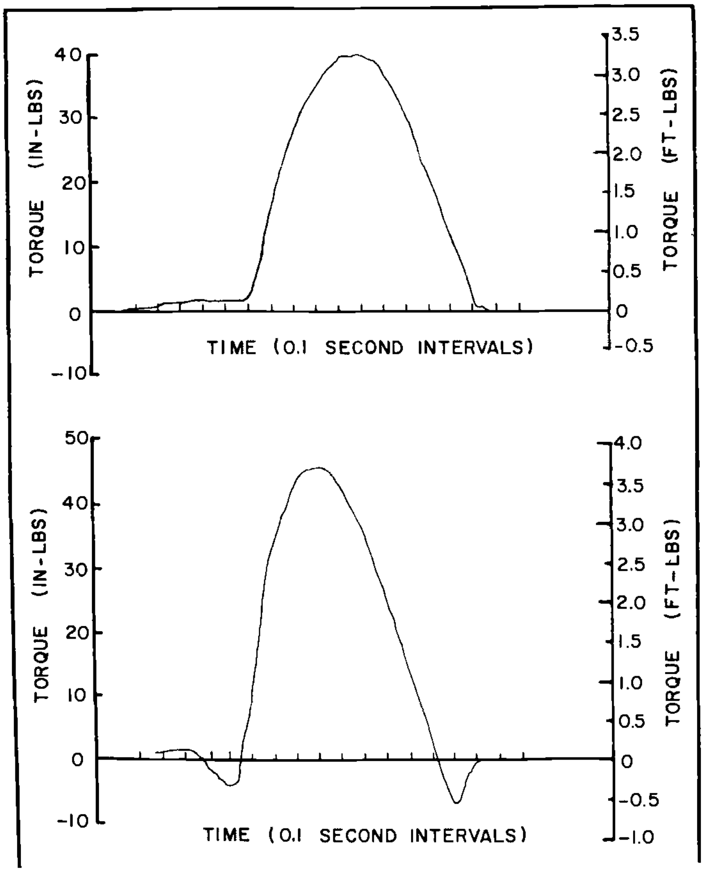

Fig. 6. Typical plots of the torque that man can exert while weightless as a function of time.

The moment of inertia of a model (consisting of "p" masses or segments) about the x-axis for the standing position is given by

$$I_{x_0} = \sum_{i=1}^{p} I_{x_{i0_{c.g.}}} + \sum_{i=1}^{p} m_i \left(y_{i0}^2 + z_{i0}^2 \right) \tag{2}$$

When the body position changes, the moment of inertia about the same axis is given by

$$I'_{x_0} = \sum_{i=1}^{p} I_{x_{i_{c.g.}}} + \sum_{i=1}^{p} m_i \left(y_i^2 + z_i^2 \right) \tag{3}$$

To find the moment of inertia about a parallel axis through the center of mass for this new position, the Parallel Axis Transfer Theorem is used

$$I'_{x_0} = I_x + M\left(\bar{y}^2 + \bar{z}^2\right) \tag{4}$$

now

$$I_x + M\left(\bar{y}^2 + \bar{z}^2\right) = \sum_{i=1}^{p} I_{x_{i_{c.g.}}} + \sum_{i=1}^{p} m_i \left(y_i^2 + z_i^2 \right) \tag{5}$$

Subtracting Eq 2 from Eq 5

$$I_x + M(\bar{y}^2 + \bar{z}^2) - I_{x_0} = \sum_{i=1}^{p} I_{x_{i\,c.g.}} + \sum_{i=1}^{p} m_i(y_i^2 + z_i^2) - \sum_{i=1}^{p} I_{x_{io\,c.g.}} - \sum_{i=1}^{p} m_i(y_{io}^2 + z_{io}^2) \quad (6)$$

Assuming the local terms do not change

$$\sum_{i=1}^{p} I_{x_{i\,c.g.}} = \sum_{i=1}^{p} I_{x_{io\,c.g.}} \quad (7)$$

and Eq 6 becomes

$$I_x = I_{x_0} - \sum_{i=1}^{p} m_i\left\{(y_{io}^2 + z_{io}^2) - (y_i^2 + z_i^2)\right\} - M(\bar{y}^2 + \bar{z}^2) \quad (8)$$

Now if only "n" masses change position, the coordinates of the "p-n" masses will remain the same and will cancel out. Then

$$I_x = I_{x_0} - \sum_{i=1}^{n} m_i\left\{(y_{io}^2 + z_{io}^2) - (y_i^2 + z_i^2)\right\} - M(\bar{y}^2 + \bar{z}^2) \quad (9)$$

In a similar manner the equations for the moments and products of inertia about the other axes are found to be

$$I_y = I_{y_0} - \sum_{i=1}^{n} m_i\left\{(x_{io}^2 + z_{io}^2) - (x_i^2 + z_i^2)\right\} - M(\bar{x}^2 + \bar{z}^2) \quad (10)$$

$$I_z = I_{z_0} - \sum_{i=1}^{n} m_i\left\{(x_{io}^2 + y_{io}^2) - (x_i^2 + y_i^2)\right\} - M(\bar{x}^2 + \bar{y}^2) \quad (11)$$

$$\bar{x} = \frac{1}{M}\sum_{i=1}^{n} m_i(x_i - x_{io}) \quad (12)$$

$$\bar{y} = \frac{1}{M}\sum_{i=1}^{n} m_i(y_i - y_{io}) \quad (13)$$

$$\bar{z} = \frac{1}{M}\sum_{i=1}^{n} m_i(z_i - z_{io}) \quad (14)$$

$m_i \equiv$ mass of the ith segment
$x_i \; y_i \; z_i \equiv$ coordinates of the center of mass of the ith segment after some change
$x_{io} \; y_{io} \; z_{io} \equiv$ coordinates of the center of mass of the ith segment before some change
$M \equiv$ total mass

and "n" is the number of segments which change position from the initial conditions. For instance, if one arm is raised from the standing position, the center of mass of the upper arm, lower arm, and hand will change. Three segments are involved so $n = 3$ and m_1 might refer to the mass of the upper arm, m_2 to the mass of the lower arm, and m_3 to the mass of the hand.

It is pointed out that Eqs 12, 13 and 14 are exact and will always yield the coordinates of the new center of mass with respect to the center of mass location for the standing position.

Up to this point, nothing has been said about products of inertia (Ixy, Iyz, and Izx). It should be realized that while in the standing position, the body axis system coincides with the principal axes of inertia and there are no products of inertia, this will not be true in general. Principal axes of inertia are defined as a set of orthogonal axes about which the products of inertia are zero. In fact, in the crouched position (Fig. 2) the principal axes are tilted forward (rotated about the y-axis) approximately 8° from the body axes. Therefore, a product of inertia exists.

From Eqs 9, 10, and 11, the moments of inertia of the model are computed for other positions. These results are compared with exact results taking the local terms into account in Table I.

It should be noted that the approximate method yields exact results for Ix, position "c," and Iy, position "b." This occurs because there is no change in the local moment of inertia terms $Ix_{c.g.}$ for position "c" and $Iy_{c.g.}$ for position "b."

EXPERIMENTAL EFFORT

Because it was desired to have some basis of comparison for the calculated moments of inertia, a test was conducted to determine experimentally the moments of

TABLE I. COMPARISON OF MOMENTS OF INERTIA FROM EXACT AND APPROXIMATE METHODS

Method	Moments of Inertia (Slug-ft.^{2})					
	I_x for Position		I_y for Position		I_z for Position	
	"b"	"c"	"b"	"c"	"b"	"c"
Exact	3.0496	12.225	2.9445	8.8430	1.0004	3.6210
Approximate	3.0845	12.225	2.9445	8.7917	0.9668	3.5356
Error	+1.14%	0.00%	0.00%	−0.58%	−3.36%	−2.36%

inertia of a living subject. The approach taken was to determine experimentally the resultant angular velocity of the body after a known torque was applied. The procedure was to have the subject apply this torque to a rigidly mounted handle during a period of simulated weightlessness and measure the resulting spin velocity.

This approach, then, required that first the nature of the torque that weightless man could exert be known. Because this is an area which is of considerable interest to many investigators,[2] it was decided that the torque input be obtained experimentally as a bonus to determining the moments of inertia.

The nature of the torque was determined under simulated zero gravity conditions on board a USAF KC-135 jet transport flying parabolic trajectories by personnel of the 6570th Aerospace Medical Research Laboratories at Wright-Patterson Air Force Base, Ohio. During periods of weightlessness, subjects grasped a rigidly mounted handle and applied, with near maximum effort, a quick torque of approximately 1-second duration. The handle was instrumented and calibrated so that the torque exerted was plotted as a function of time. As a result of the torquing, the subjects began to spin; the angular velocity of the spin was measured from motion picture films of the experiment.

Two typical torque vs. time plots are shown in Figure 6. As can be readily seen, the curves closely resemble a half sine wave. Assuming that the torque input varies exactly as a half sine wave, then

$$L(t) = L_m \sin\frac{\pi t}{T} \tag{15}$$

where $L(t)$ = torque as a function of time
L_m = maximum torque achieved
t = time
T = period of torque application

For rotation about one of the principal axes of inertia we have

$$I\dot{\omega} = L(t) = L_m \sin\frac{\pi t}{T} \tag{16}$$

where
I = moment of inertia
$\dot{\omega}$ = angular acceleration

Assuming I is constant, we have after integrating

$$I\omega = \frac{L_m T}{\pi}\left(1 - \cos\frac{\pi t}{T}\right) \tag{17}$$

and at $t = T$

$$I\omega_T = \frac{2L_m T}{\pi} \tag{18}$$

$$I = \frac{2L_m T}{\pi \omega_T} \tag{19}$$

where ω_T = angular velocity at $t = T$

Two subjects were tested and the analytical results fell within ± 10 per cent of the experimental results.

SUMMARY

The mathematical model developed to represent weightless man is based on the biomechanical properties of the human body. Because there are no methods of determining many of these properties accurately from a living subject, statistical data are used which depend on the total body weight. Body dimensions, however, can be measured for any given living subject.

An analysis of the 14-segment model reveals that the transfer terms are the most important parts of the total moments of inertia, but that the local inertia terms of the torso are quite significant for some axes. Because the transfer term "md^2" depends upon the square of the distance between the mass center and the inertia axis, it is more sensitive to variations in distance than to mass variations. Hence, a model based on anthropometry of a given subject will reflect the dynamic response characteristics of that subject.

The system of equations presented for the body mass center and moments of inertia is based on the knowledge of the mass center and the three principal moments of inertia for a given position. These quantities can be computed as in Reference 8, or determined experimentally. The system of equations then yields these quantities for any other position.

A useful application of these equations would be in conjunction with an extra-vehicular operations simulator/trainer for the astronaut. Here the equations would present the human parameters as inputs into the dynamic response characteristics of a personal propulsion and stabilization system being simulated. If a full scale trainer is used such that the astronaut is suspended free to move all limbs, linkages could be attached to the limbs so that limb position could be constantly fed into a computer. The necessary human parameters could be represented as a function of time during training maneuvers, and programmed into the simulator.

While the experiment to determine moments of inertia was crude, the information obtained on the magnitude, duration, and variation of the torque output of weightless man should be useful to many researchers. Most significant was that the torque varied as a half sine wave. Further, it was noted that the subjects could only exert 2/3 of their peak 1-g torque while weightless.

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