Requirements for Experimental Zero Gravity Parabolas

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THE BASIC physics, equations of motion of a body during subgravity states as well as techniques for producing zero and subgravity, were presented in earlier reports.[1-5] It has become apparent, however, that a more simplified presentation of the principles involved in experimental production of zero gravity trajectories is necessary for the less mathematically-inclined pilots and flight surgeons. Many of them have expressed a desire for graphs which, without calculation, would answer the following question: To produce a specific number of seconds of zero gravity, what must be the aircraft entry velocity, angle of climb at entry, and maximum altitude attained during this parabolic maneuver?

For practical purposes, the term *parabola* has for some time been applied to the zero gravity trajectories presently produced by jet aircraft. As long as the linear dimensions of the trajectory are small in comparison to the radius of the earth, the latter can be considered a plane with the lines of gravitational force being parallel and perpendicular to that plane. In this case the elliptic arc degenerates to a parabola. For this reason present-day zero-g-trajectories closely approximate parabolas, and the term parabola, therefore, will be used throughout this paper. The mathematical analysis which follows is also based on the plane gravitational field model using the equations of the parabola.

The law of gravitation is probably the most universal law thus far derived; and since the acceleration of the attracted body is independent of its mass, then

$$a = Gm/d^2 \qquad (1)$$

where a is the acceleration of a body falling toward the earth, G is the constant of gravitation, m is the mass of the earth, and d is distance from the center of the earth.

Because for the present and immediate future we will be involved in producing experimental zero gravity trajectories below 200,000 feet, we can assume gravitational acceleration to be a constant. For ease in remembering, 32.2 ft./sec.^{2} was selected.

Sustained (many seconds) production of a state of zero gravity can at present be achieved only by flying a high-speed aircraft in such a way that all forces are kept in equilibrium. The two principal factors determining the shape of the flight profile and duration of exposure to weightlessness are the *minimum* aircraft velocity without stall and the *maximum* aircraft velocity capable of being achieved by the aircraft.

It must be pointed out that in zero G flights stalling conditions are different as compared to conventional

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flight maneuvers, because the plane is weightless and is, therefore, in no need of aerodynamical support. On the other hand, a minimum speed is required to execute the delicate control through the parabola. For this reason the stalling speed is taken as the minimum aircraft velocity required for effective control.

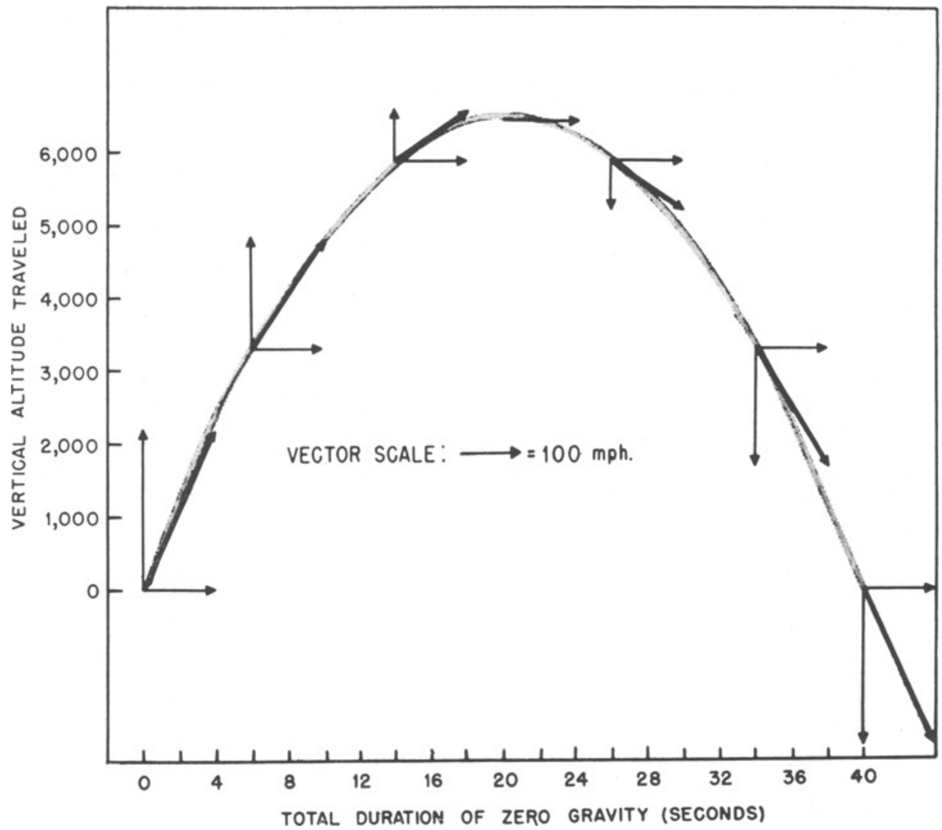

Fig. 1. Velocity vector components encountered in a zero gravity parabola.

Because at all times during the zero gravity trajectory the horizontal velocity must remain constant, there is zero acceleration in the horizontal plane. At the peak of the "parabola" the vertical velocity coordinate becomes zero and at this time the only component of velocity remaining is the constant horizontal velocity (Fig. 1). Therefore, the minimum speed of the aircraft during the entire zero gravity maneuver will occur at the peak or apex of the "parabola," that is, at the maximum altitude achieved. For any particular aircraft the stalling speed is a constant for any given altitude and atmospheric density (the aircraft is in straight and level flight at the instant of maximum altitude achievement). Consequently, the minimum air speed attainable without stall is a function of: (1) air density and, therefore, altitude; (2) the weight of the aircraft; and (3) the area of the wings. Then the *shape* of the flight profile curve can

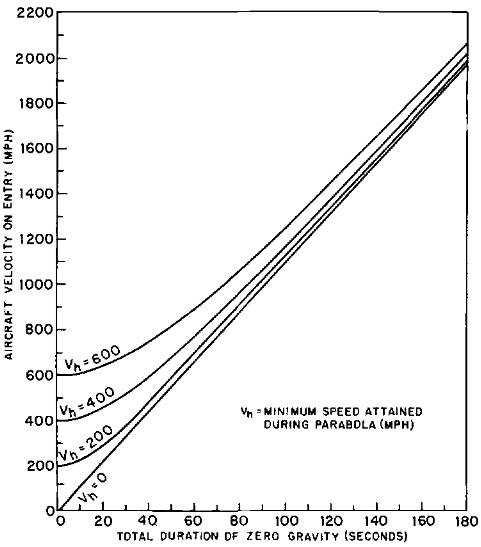

Fig. 2. The parabolic entry velocity required for zero gravity flight.

be determined prior to flight by calculating the value of this constant horizontal velocity.

The entry speed and angle of entry into the maneuver are the primary determining factors in duration of zero gravity. To maintain all forces, accelerative and inertial, in balance, the aircraft must lose 21.9 mph (32.2 ft. per sec.) of vertical velocity during each second of the upward limb of the flight and gain this much velocity every second on the downward limb of the maneuver. On the upward limb of the "parabola," the vertical velocity coordinate will decrease to zero at the apex. After this the vertical velocity coordinate will begin to point in the opposite direction, increasing in length 21.0 mph during each second of the downward limb. The actual aircraft air speed is the vectorial sum of the constant horizontal velocity and the constantly-changing vertical velocity.

Where t is the time from the onset of the trajectory (in seconds) to any point, x along the path, and T is the total duration of the zero gravity parabola, and if

$t \lesseqgtr T/2,$
then the time from the
$\text{apex} = T/2 - t$ (2)
and
$t > T/2,$
then the time from the
$\text{apex} = t \lesseqgtr T/2$ (3)

Then let V_x=velocity (vectorial sum) at any point x on the flight path

V_v=vertical velocity vectorial component at point x
V_h=constant horizontal vectorial component of velocity

And since at the apex of the trajectory $V_v = O$, and V_v changes 32.2 ft./sec. or 21.96 mph each second on each side of the apex, then

$$V_v = \pm\ (21.96)(t - T/2) \text{ mph} \quad (4)$$

and

$$V^2_v = (21.96t - 10.98\ T)^2 \quad (5)$$

And since V_x is the vectorial sum of V_v and V_h, then

$$V^2_x = V^2_v + V^2_h \quad (6)$$

Substituting from equation 5

$$V_x = [(21.96t - 10.98T)^2 + V^2_h]^{1/2} \quad (7)$$

At the onset of the trajectory $t = O$ and

$$V_o = (120.6\ T^2 + V^2_h)^{1/2} \quad (8)$$

where V_o is the vectorial sum of velocities at entry. Equation 8 is expressed in graphic form by Figure 2 in which aircraft entry velocity is determined knowing the duration of zero gravity desired and the stalling speed of the aircraft. Angle of climb above or dive below horizon can be determined since the vertical vectorial component comprises the opposite and the vectorial sum comprises the hypotenuse of the right-angle triangle.

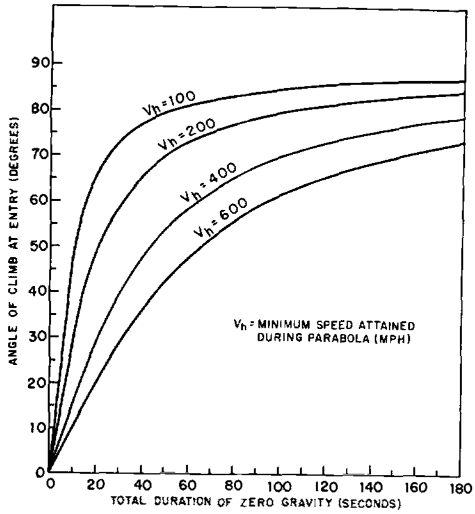

Fig. 3. Angle of climb required at entry for zero gravity flight.

Therefore,

$\sin O_x = V_v/V_x$ (9)
where O_x is the angle of climb or dive at point x on the trajectory.

And since from equation (4)

$V_v = 10.98T$ at entry (10)

Then

$$\sin O_o = \frac{10.98T}{V_o} \text{ at entry.} \quad (11)$$

Figure 3 expresses in graphic form angle of climb at entry into the trajectory as a function of the duration of zero gravity desired and the minimum speed attained during the maneuver by the aircraft.

The average vertical vectorial component velocity from onset to apex will be the vertical vectorial component velocity at one-half the time to the apex from onset,—that is, at $t=T/4$ time.

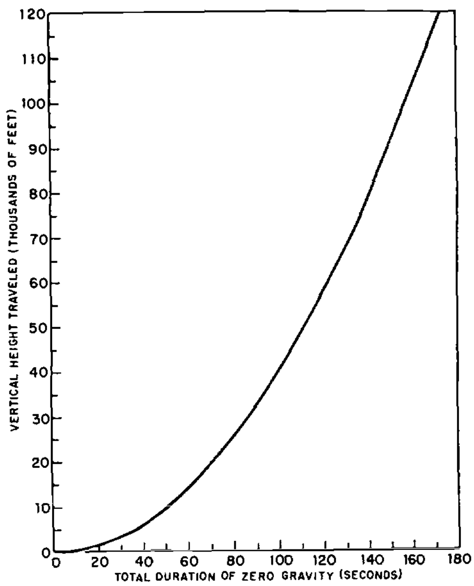

Fig. 4. The influence of total duration of zero gravity on the vertical height traveled during parabolic flight.

Therefore, from equations 2, 3, and 4

$V_v(T/4) = 21.96(T/2-t) = 5.490T$ mph (12)

Converting mph to ft./sec. units

$V_v(T/4) = 8.052T$ ft./sec.

If H_o is the altitude at the onset of the maneuver and H_{max} is the maximum altitude achieved during the trajectory, then

$H_{max} = H_o + (V_{vmean})$ (time from onset to apex) (13)

and from equations 12 and 2

$H_{max} = H_o + 4.026\, T^2$ in feet (14)

From the formula for vertical altitude traveled during the maneuver ($4.026T^2$), the curve shown in Figure 4 is derived.

SUMMARY

In simplified form, graphs are presented which allow rapid determination, without calculation, of parabolic entry velocity, angle of climb at entry, and vertical altitude traveled during the trajectory as a function of the total duration of zero gravity and minimum speed attained during the parabola (determined by stalling speed) by the experimental aircraft.

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The Upward Extension of Aviation Medicine

With the rocket at our disposal, astronautics, or the conquest of space, is in sight. In fact, we have already penetrated deeply into space immediately around us with unmanned research vehicles. And we have had manned flight operations of the space equivalent variety for the past several years. Manned space operations of the satellite type, lunar operations, and interplanetary and planetary operations are the logical further developments. All of these missions which will carry man completely out of the atmospheric environment so vital to him will pose medical problems unheard of in human history. That new medical field which studies the human factors involved and pertinent protective measures required in the penetration of space is space-flight medicine or, briefly, *space medicine* —a branch or an extension of aviation medicine. It is the mission of space medicine to contribute to the development, safety, and efficiency of manned space flight or astronautics. Space medicine can, therefore, also be called bioastronautics following a motto *astronautico subvenimus*.—HUBERTUS STRUGHOLD: Interrelations of Space Medicine with Other Fields of Science, *Aeronautical Engineering Review*, April, 1958.